**Curriculum**

**IB Math HL Y1**

**Course Overview**

This is a two-year course designed to prepare students with a strong background and keen interest in mathematics for the International Baccalaureate Diploma Program, a rigorous pre-university course of studies. Many of the topics for the Higher Level course are similar to the topics covered in the Standard Level course but will be examined at a much deeper level than the Standard Level. A TI-83+ or TI-84+ is required for this course. In the first year, students will study five of the eight topics that are to be covered during the two-year course and complete some portfolio assignments that may be used as the internal assessment component of the students’ IB score at the end of the course. The five topics include Algebra, Functions and Equations, Circular Functions and Trigonometry, Matrices, and Vectors. The portfolio assignments will be of varying difficulty and will cover different areas of the curriculum. The purpose of the portfolio is to give students the opportunity to study a particular topic in greater detail. It will also provide the students with the opportunity to communicate effectively with sound mathematical writing.

**Department Standards**

Students will be able to comprehend mathematical concepts.

Students will apply mathematical procedures accurately, efficiently, and appropriately.

Students will be able to formulate, represent, and solve mathematical problems.

Students will develop logical mathematical thought and precise mathematical communication.

**Benchmarks**:

Students will be able to:

produce algebraic strategies for solving problems;

explain the notion of function as a unifying theme in mathematics and apply functional methods to a variety of mathematical situations;

apply concepts in circular functions and trigonometry to check identities, evaluate equations and triangles;

apply operations to matrices and implement solutions of systems of equations;

apply operations to vectors in two and three dimensions and implement solutions to problems involving points, lines, and planes;

understand and explore formal method of proof, including induction.

**Performance Indicators**

First Quarter

Students will be able to:

recognize graphs, vertices and edges;

determine if edges and vertices are adjacent;

determine if graphs are simple, connected and/or complete; define bipartite and planar graphs;

define a tree;

utilize weighted graphs;

find subgraphs and complements of graphs;

determine if graphs are isomorphic;

find walks, trails, paths, circuits and cycles in graphs;

define and state the conditions for the existence of Hamiltonian paths and cycles in a graph;

define and state the conditions for the existence of Eulerian trails and circuits in a graph;

create and use an adjacency matrix;

create and use a cost adjacency matrix;

use Prim, Kruskal and DijkstraÆs algorithms;

determine the shortest route around a

weighted graph going along each edge at least once;

determine the Hamiltonian cycle of least

weight in a weighted complete graph;

determine the upper and lower bounds of the traveling salesman problem;

identify domain, range and specific function values for a given function or graph of a function;

find the composite of two given functions; evaluate a composite function;

identify the identity and reciprocal functions as equations and on graphs;

find the inverse of a function from an equation and through graph symmetry;

graph absolute value functions;

graph functions without using a GDC;

use a GDC to effectively graph functions;

identify horizontal and vertical asymptotes;

identify inequalities in one variable, using their graphical representation;

solve g(x)>f(x), where f and g are linear or quadratic;

use the absolute value sign in inequalities;

determine whether a sequence is arithmetic, geometric or neither

supply the missing terms of a sequence;

find a formula for the nth term of an arithmetic or geometric sequence;

find specific terms of an arithmetic or geometric sequence;

represent series using sigma notation;

find the sum of finite arithmetic and geometric series;

apply sequence and series knowledge in applications of compound interest and population growth;

find the sum of an infinite geometric series; define logarithmic functions;

relate logarithmic and exponential functions;

apply the basic properties of logarithms;

change the base of a logarithm;

use common logarithms in solving equations;

use exponential and logarithmic functions to solve growth and decay problems; and

define and use the natural logarithm function.

Second Quarter

Students will be able to:

graph and analyze the vertical shift, horizontal shift, reflection and stretch of the graph of equations;

find the vertices and axis of symmetry of quadratic equations in vertex form and general form;

analyze a quadratic equation, draw its graph, and find its maximum and minimum value;

write a quadratic equation or function using information about the roots and/or the graph;

solve quadratic equations using various methods including factoring, quadratic formula or completing the square;

use the discriminant to determine the nature of the roots of a quadratic equation;

identify the real part and the imaginary part of a complex number;

find the conjugate of a complex number;

find sums, products, and quotients of complex numbers;

find solutions of real quadratics with <0;

find polynomial equations with real coefficients given conjugate roots;

add, subtract, multiply, and divide polynomials;

find polynomial equations with real coefficients given one, two, or more roots;

find zeros of polynomials;

understand and apply the factor and remainder theorems towards finding solutions of polynomial equations and inequalities;

graph polynomials;

identify the graphical significance of repeated roots;

find permutations and combinations;

use the binomial coefficient to find combinations;

expand powers of binomials; and

use the binomial theorem to find a term of a binomial expansion.

Third Quarter

Students will be able to:

make conjectures about given statements;

use the principal of mathematical induction to prove given propositions;

identify the parts of a circle;

convert from radians to degrees and degrees to radians;

work with radians expressed as multiples of  or as decimals;

find the length of an arc;

find the area of a sector;

define cos and sin in terms of the unit circle;

find possible values of cos given sin, without finding ;

define tan as sin/cos;

define sec, csc, and cot;

use the Pythagorean identities;

use and prove the compound angle identities;

use and prove the double angle identities;

find possible values of other ratios (like sin2), given sin, without finding ;

recognize and work with the circular functions sin(x), cos(x), and tan(x) by: identifying their domains and ranges, observing their periodic nature, and drawing their graphs;

find functions in the form f(x)=asin(b(x+c))+d given the graphs of those functions;

sketch graphs of functions of the form f(x)=asin(b(x+c))+d;

apply functions of the form f(x)=asin(b(x+c))+d to height of tide and Ferris wheel problems;

use inverse functions y=arcsinx, etc, know their domains and ranges, and be able to graph them;

solve trigonometric equations given a finite interval;

transform equations using trigonometric identities and factorization, both analytically and graphically;

recognize trigonometric functions in quadratic form, solve them, and interpret their graphs;

use the Pythagorean theorem and trigonometric ratios to solve right triangles;

use the cosine rule to solve triangles and understand when it is applicable;

use the sine rule to solve triangles and understand when it is applicable;

remember there is an ambiguous case when using the sine rule and recognize that there may be two solutions when solving for ;

find the area of a triangle using A=(1/2)absinC, and apply it to problems in real-life situations;

identify the element, row, column, and order of a matrix;

use matrices to store data;

determine equality of matrices;

add and subtract matrices;

multiply a matrix by a scalar;

multiply matrices;

define identity and zero matrices;

find the determinants of square matrices;

define the terms singular and non-singular matrices;

calculate determinants for 2X2 and 3X3 matrices;

use the result: detAB=detAdetB;

find the inverse of a 2X2 matrix;

describe the conditions for the existence of the inverse of a matrix;

obtain the inverse of a 3X3 matrix using a GDC;

solve systems of linear equations (with a maximum of three equations in three unknowns); and

describe the conditions for the existence of a unique solution, no solution and an infinity of solutions using inverse matrices and row reduction.

Fourth Quarter

Students will be able to:

represent vectors as displacements in the plane and in three dimensions;

find the distance between points in three dimensions;

recognize the components of a vector and be able to represent them in columns;

find the sum and difference of two vectors using both algebraic and geometric approaches;

recognize the difference of v and w is v-w=v+(-w);

identify and represent the zero vector and the vector -v both algebraically and geometrically;

represent multiplication by a scalar geometrically;

find the magnitude of a vector;

change from unit vector form to a base/component vector form and vice versa;

use correct notation to represent position vectors;

find the scalar product of two vectors;

recognize that the scalar product is also known as the "dot product" or "inner product";

apply the algebraic properties or the scalar product;

identify parallel vectors;

show two vectors are parallel;

know that for non-zero parallel vectors v.w=▒|v||w|;

show that two vectors are perpendicular, v.w=0;

know that for non-zero perpendicular vectors v.w=0;

find the angle between two vectors;

write vector equations of lines in the plane and in three-dimensional space;

use parametric form and Cartesian form for equations of lines;

find the angle between two lines;

define and distinguish between coincident, parallel, intersecting and skew lines;

find points of intersection;

find the vector product of two vectors;

know that the vector product is also known as the cross product;

make a geometric interpretation of |v╫w| relating to areas of triangles and parallelograms;

use vector equations of a plane;

use a normal vector to obtain the form ròn=aòn;

use Cartesian equations of a plane.

find intersections of: a line with a plane; two planes; three planes;

use inverse matrix method and row reduction for finding the intersection of three planes;

find the angle between: a line and a plane; two planes;

recall that three planes may intersect in a point, or in a line, or not at all.

identify the modulus and argument of a complex number;

represent complex numbers in modulus-argument form;

recall that z=r(cos+isin) can be written as z=re^(i) and z=cis;

use the complex plane;

know that the complex plane is also known as the Argand diagram;

apply De MoivreÆs theorem;

prove De MoivreÆs theorem using mathematical induction;

find powers and roots of complex numbers;

use the definition of derivative as a limit for differentiation of polynomials, and for justification of other derivatives;

use both function and differential notation for the derivative;

understand the derivative as a function representing the gradient at a point;

find the derivative of a sum and a real multiple of a polynomial function;

recognize that the derivative of a constant is zero;

use the chain rule to find the derivative of composite polynomial functions, f(g(x));

use the product rule to find the derivative of the product of polynomial functions, f(x)\*g(x);

use the quotient rule to find the derivative of the quotient of polynomial functions, f(x)/g(x);

find the equations of tangents and normals; recognize both forms of notation;

appreciate the existence of higher derivatives; identify increasing and decreasing functions;

find local maximum and minimum points; test for the maximum or minimum using change of sign of the first derivative;

test for maximum and minimum by using sign of second derivative; use the first and second derivative in

optimization problems such as profit, area, and volume;

differentiate an expression with respect to any variable;

find the derivative of exponential equations in base e and a;

find the derivative of logarithmic equations in base e and a;

find the derivative of.sin x , cos x , tan x;

find the derivative of reciprocal trigonometric functions sec x , csc x , cot x;

find the derivative of inverse trigonometric functions arcsin x , arccos x , arctan x;

find the local maxima and minima of trigonometric functions;

derivative interpreted as a gradient function and as rate of change.functions; and

use of the chain rule in related rates.

**Assessments**

First Quarter

Unit Tests on Core Topics

Mathematical Investigation Portfolio Piece

Second Quarter

Unit Tests on Core Topics

Mathematical Modeling Portfolio Piece

First Semester Exam

Third Quarter

Unit Tests on Core Topics

Mathematical Investigation Portfolio Piece

Mathematical Modeling Portfolio Piece

Fourth Quarter

Unit Tests on Core Topics

Mathematical Investigation Portfolio Piece

Second Semester Exam

**Core Topics**

First Quarter

Graph Theory

Functions

Sequences and Series

Exponentials

Logarithms

Second Quarter

Graphing and Transforming Functions

Quadratic Equations and Functions

Complex Numbers and Polynomials

Counting and the Binomial Expansion

Third Quarter

Mathematical Induction

The Unit Circle and Radian Measure

Non-Right Angled Triangle Trigonometry

Advanced Trigonometry

Matrices

Fourth Quarter

Vectors in Two and Three Dimensions

Complex Numbers

Lines and Planes in Space

Introduction to Differential Calculus

**Specific Content**

First Quarter:

Graphs, vertices, edges. Adjacent vertices, adjacent edges.

Simple graphs; connected graphs; complete graphs; bipartite graphs; planar graphs, trees, weighted graphs.

Subgraphs; complements of graphs.

Graph isomorphism. Walks, trails, paths, circuits, cycles.

Hamiltonian paths and cycles; Eulerian trails and circuits.

Adjacency matrix. Cost adjacency matrix.

Graph algorithms: PrimÆs; KruskalÆs; DijkstraÆs.

ôChinese postmanö problem (ôroute inspectionö).

Travelling salesmanö problem

Algorithms for determining upper and lower

bounds of the travelling salesman problem.

Concept of a function f(x): domain, range; image (value).

Composite functions f(g(x)); identity function.

Inverse function. The graph of a function; its equation y = f(x).

Function graphing skills: use of GDC to graph a variety of functions; investigation of key features of graphs; solutions of equations graphically.

The graph of the inverse function as the reflection in the y = x of the graph of y = f(x).

The graph of y=1/x from y=f(x).

The graphs of the absolute value functions, y=|f(x)| and y=f(|x|).

The reciprocal function f(x) = 1/x, x  0: its graph; its self inverse nature.

Inequalities in one variable, using their graphical representation.

Solution of g(x)>f(x), where f, and g are linear or quadratic.

Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series.

Sigma notation.

Exponents and logarithms.

Laws of exponents; laws of logarithms.

Change of base.

The function: y=a^x, a>0.

The inverse function: y=log(base a)x, x>0.

Graphs of y = a^x and y = log(base a) x.

Solution of a^x = b using logarithms.

The exponential function: y=e^x.

The logarithmic function: y=ln x; x > 0.

Second Quarter:

Transformations of graphs: translations; stretches; reflections in the axes.

The quadratic function f(x) = ax^2 + bx + c: its graph.

Axis of symmetry x = -b/(2a).

The form f(x) = a(x - h)^2 + k: vertex (h, k).

The form f(x) = a(x - p)(x - q): x intercepts (p, 0) and (q, 0).

The solution of ax^2 + bx + c = 0, a  0.

The quadratic formula.

Use of the discriminant  = b^2 - 4ac.

Complex numbers: the number i; the terms real part, imaginary part, conjugate.

Cartesian form, z=a+ib.

Sums, products, and quotients of complex numbers.

Conjugate roots of polynomial equations with real coefficients.

Polynomial functions.

The factor and remainder theorems, with application to the solution of polynomial equations and inequalities.

Counting principles, including permutations and combinations.

The binomial theorem: expansion of (a + b)^n, n {Natural Number}.

Third Quarter:

Proof by mathematical induction.

Forming conjectures to be proved by mathematical induction.

The circle: radian measure of angles; length of an arc; area of a sector.

Definition of cos and sin in terms of the unit circle.

Definition of tan as sin/cos.

Definition of sec, csc and cot.

Pythagorean identities.

Compound angle identities.

Double angle identities.

The circular functions sin(x), cos(x), and tan(x): their domains and ranges; their periodic nature; and their graphs.

Composite functions of the form f(x)=asin(b(x+c))+d.

\*\*Here?\*\*The inverse functions of sin(x), cos(x), and tan(x); their domains and ranges; their graphs.

Solution of trigonometric equations in a finite interval.

Use of trigonometric equations in a finite interval.

Use of trigonometric identities and factorization to transform equations.

Solutions of triangles.

The cosine rule.

The sine rule.

Area of a triangle as (1/2)absinC.

Definition of a Matrix: the terms "element", "row", "column", and "order".

Algebra of Matrices: equality; addition; subtraction; multiplication by scalar.

Multiplication of Matrices.

Identity and zero matrices.

Determinant of a square matrix.

Calculation of 2X2 and 3X3 determinants.

Inverse of a matrix: conditions for its existence.

Solution of systems of linear equations (a maximum of three equations in three unknowns).

Conditions for the existence of a unique solution, no solution and an infinity of

solutions.

Fourth Quarter:

Vectors as displacements in the plane and in three dimensions.

Components of a vector; column representation.

Algebraic and geometric approaches to the following topics: the sum and difference of two vectors; the zero vector; the vector, -v; multiplication by a scalar, kv; magnitude of a vector, |v|; unit vectors; base vectors i, j, and k; position vectors OA=a.

The scalar product of two vectors, v\*w = |v||w| cos; v\*w= v1w1 + v2w2 + v3w3.

Algebraic properties of the scalar product.

Perpendicular vectors; parallel vectors.

The angle between two vectors.

Vector equation of a line r=a+i.

The angle between two lines.

Coincident, parallel, intersecting and skew lines, and distinguishing between these cases.

Points of intersection.

The vector product of two vectors.

The determinant representation.

Geometric interpretation of |v╫w|.

Vector equation of a plane, r=a+b+c.

Use of a normal vector to obtain the form ròn=aòn.

Cartesian equation of a plane, ax+by+cz=d.

Intersections of: a line with a plane; two planes; three planes.

Angle between: a line and a plane; two planes.

Complex numbers: modulus and argument;

Modulus-argument form, z=r(cos+isin).

The complex plane.

DeMoivreÆs theorem.

Powers and roots of a complex number.

IF EXTRA TIME OR SUMMER STUDY:

DIFFERENTIAL CALCULUS

Informal ideas of limit and convergence The gradient function

Differentiation rules- sums, multiples, the chain rule, products and quotients.

Tangents and Normals

The second derivative. Optimization

Implicit differentiation

FURTHER DIFFERENTIATION

Derivatives of exponential and logarithmic functions.

Derivatives of circular functions

Maxima/minima with trigonometry

Related Rates

**Resources**

TI-84 Plus

Mathematics for the International Student: Mathematics HL Core

Haese & Harris Publications 2nd edition

Mathematics for the International Student: Mathematics HL Options

Haese & Harris Publications

SMARTBoard software